JPMC QR Mentorship Program 2024 Case Study

Monte Carlo Simulation and Option Pricing

**Suggested Format**

This document contains case study on Monte Carlo Simulations and Option Pricing with three questions. A question might have multiple sub-parts and we suggest the solution format for each alongside the question to guide you through the challenge.

Python is the preferred programming language but in case you are not familiar with Python please use the language of your choice. We suggest you not import direct libraries for Monte Carlo or Black-Scholes as you will be learning the details about these topics through the actual implementation.

You are free to use online resources and discuss with your mentor and fellow mentees to improve your understanding of the problem statement.

As you work on the problems, you will discuss your progress with your mentor and ask for feedback on your solutions. We will not grade your submissions however we expect you to be proactive in solving problems, discussing ideas, and soliciting feedback to learn and grow.

**Solution & Question Submissions**

Please submit your final solution to ***QR\_NA\_Mentorship\_Program\_External@jpmchase.com*** with“***JPMC QR Mentorship Case Study Submission <FirstName\_LastName>”*** in your email subject.

Office hours and support for this Case Study will end **Aug 19** so we encourage you to complete this case study within the allotted time.

You are welcome to submit your case-study related questions to ***QR\_NA\_Mentorship\_Program\_External@jpmchase.com.*** Please use the format “***JPMC QR Mentorship Case Study Question <FirstName\_LastName>”*** in your email subject.

**Monte Carlo Simulation**

Monte Carlo Simulation (or Method) is a probabilistic numerical technique used to estimate the outcome of a given, uncertain (stochastic) process. This means it’s a method for simulating events that cannot be modeled implicitly (using deterministic models). This is usually a case when we have random variables in our processes.

**How to perform simulations in real time**

On an imaginary Island where the total population is 50K, we need to compute the average height of the inhabitants. Let’s call this variable H (in non-specified units); and we know that it follows uniform distribution, i.e. h ~ U[5, 6].

This can be solved analytically as we know that the average of uniform variable = (higher bound + lower bound)/2 which in this case yields 5.5.

Simulations can also come handy when dealing with random variables as we will see in this case. When a closed form solutions doesn’t exist, which is very common in finance, we rely on numerical methods to estimate the target variable.

In the following example, we can simulate the heights of the inhabitants and then take the mean of the heights. Where our input variable is the uniformly distributed variable.

*Code:*

*>>> ### The following command, np.random.uniform will generate 50000 uniform random numbers between 5 and 6*

*>>> import numpy as np*

*>>> np.random.uniform(low = 5.0, high = 6.0, size = 50000).mean()*

*5.4983446869547556*

*>>> np.random.uniform(low = 5.0, high = 6.0, size = 50000).mean()*

*5.5004299438854565*

**Expected Value**

The expected value formula is the probability of an event multiplied by the value of the event (an event can be anything from the number of time it appears to the dollar value associated with it):

E(f(x)) = Summation of (P(x) \* f(x)) for all x

e.g. You are playing a dice game, where if you roll a six you win $100 and you lose $10 if anything else happens.

X: the event that 6 shows up, P(x) = 1/6

Y: the event that any number other than 6 shows up, P(y) = 5/6

So the expected winnings for the game will be: 1/6\*100 + 5/6\*(-10) = 8.33

***Question 1: Probability & Monte Carlo Simulation***

Consider a company “XYZ capital” currently trading at $100 (at time = t0) on the National Stock Exchange of Vol-land and it moves up by $1 or moves down by the same amount with equal probability at each minute.

1. What will be the expected value of stock price after:
2. 1 minute
3. 10 minutes
4. 1 hour
5. 1 month

Are all the values same? Please explain on why they should or should not be same

1. What will the probability of the stock price going to $102 before going to $96,
2. Solve the probability analytically.
3. Find the probability using Monte Carlo method. How many paths will get you to the converged value? Plot the estimated Value vs Number of Paths

NOTE: Tolerance limit between analytical and numerical solution is 10^-2

*Suggested Solution Format: Please attach your code with comments, describe your analytical solution as well.*

**Present Value**

Present value (PV) is the current value of a future sum of money or stream of cash flows given a specified rate of return - assuming periodic compounding

PV : Present Value

FV : Future Value

r : rate of return over a single period

n : number of periods

**Option**

An option is a contract which conveys to its owner, the holder, the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified price (known as strike price) on or before a specified date (known as expiration date).

**Call Option**

Call options are financial contracts that give the option buyer the right but not the obligation to buy a stock, bond, commodity, or other asset or instrument at a specified price within a specific time window.

**Pay-off of a call option**

Max(0, spot price - strike price )

**Pricing an Option**

Theoretically the price of an option should be the present value of expected payoff of the option at expiry. There are several ways to price an option

* Using closed form analytical solutions, such as Black-Scholes model
* Using numerical techniques such as Monte-Carlo simulations, Binomial tree pricing, finite differencing.

For this exercise we will be looking at Black-Scholes and Monte Carlo simulations.

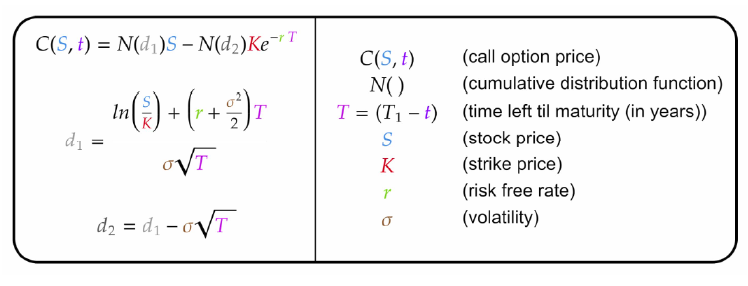
To define an option contract, we need the following parameters:

1. Strike Price: K
2. Spot Price: S(t) at time t
3. Implied Vol: V (for this exercise we will assume it to be constant)
4. Interest Rate: r (risk free rate)
5. Time To expiration: T

**Pricing a call option**

Here is the Black-Scholes Formula

(Wiki: <https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_model>)



***Question 2: Pricing a Call Option analytically using Black Scholes formula***

Consider a company “XYZ capital” currently trading at $200 (at time = t0) on the National Stock Exchange of Vol-land. We have a **call** option contract with the following parameters:

1. Strike: $180
2. Time To expiration: 30 days (1/12th of a year)
3. Implied Vol: 15%
4. Interest Rate: 2%
5. Current Stock Price: $200

Compute the value of the above-described option contract using Black Scholes formula. Please write a generic function to calculate Black-Scholes price and call the function with the inputs above.

*Suggested Solution Format: Please attach your code with comments, describe your steps to solve the problem as well.*

*Note that you can rely on stats tools & packages for this exercise but are not allowed to directly import the black Scholes library if available on the open-source platform.*

***Question 3: Pricing a Call Option using Monte Carlo Simulation***

As we discussed above the price of an option should be the present value of expected payoff of the option. Now consider a case where we know how the underlying asset moves with time and we also know how to compute the payoff of an option.

The classic Black-Scholes-Merton model assumes that stock prices follow a lognormal distribution based on the principle that asset prices cannot take a negative value and are bounded by zero. This can be represented in the following stochastic differential equation:

Where is a Brownian motion, which follows a standard normal distribution ,

Using the Euler Discretization Scheme to solve the equation above, we obtain the expression below that describes the underlying stock movement over time:

1. Please write a function to compute the price of a call option with the same parameters in Question 3 but using Monte Carlo simulation techniques.
2. Plot the price of the option vs number of paths.

NOTE: You will have to define another variable time step (T- (T-1)) to discretize the process and simulate your paths. Each path will consist of list of stock prices, and please take at least 240 steps for a single path.

*Suggested Solution Format: Please attach your code with comments, describe your steps to solve the problem as well.*